Fundamentals of Communications Engineering

Department of Communications Engineering, College of Engineering, University of Diyala, 2016-2017

Class: Second Year
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Room: Comm-02

Second Lecture

Deterministic and Random Signals :-

* If any signal can be completely specified as a specified function of time, it is called deterministic signal.
* All the previous functions in this lecture are deterministic signals.
* Random signals, are those signals which can not be specified as a function of time, they are random in nature.

Analog and Digital Signals :-

* If the signal's magnitude takes any real value, it will be called analogue signal (may be it is discrete-time).
* If the signal's magnitude takes finite number of values, it can be called digital signal.

Power and Energy signals

* signals can contain energy or power.
* The signal can be considered as an energy-type if and only if its energy value is finite.

$$E_g = \lim_{T\to\infty} \int_{-T/2}^{T/2} |g(t)|^2 dt \quad \text{———} (38)$$

or

$$E_g = \int_{-\infty}^{+\infty} |g(t)|^2 dt \quad \text{———} (39)$$

* if the signal is real then $|g(t)|^2$ will be $g^2(t)$.

* The signal can be considered as a power-type if and only if $0 < P_g < \infty$

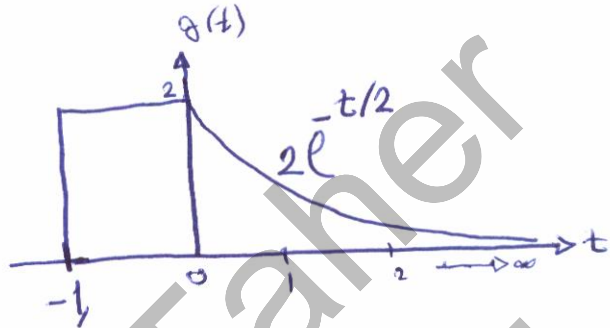
$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 \, dt \quad \text{———} (40)$$

* if $g(t)$ is real then $|g(t)|^2$ will be $g^2(t)$.
* periodic signal is a power-type.
* Aperiodic signal is an Energy-type.

EX1 sketch and find the type of the signal

$$g(t) = \begin{cases} 2 & -1 \le t \le 0 \\ 2e^{-t/2} & 0 < t \le \infty \end{cases}$$

and find its value.



solution

g(t) will reaches zero as $t \to \infty$

thus, it is not periodic, hence, it is an energy signal.

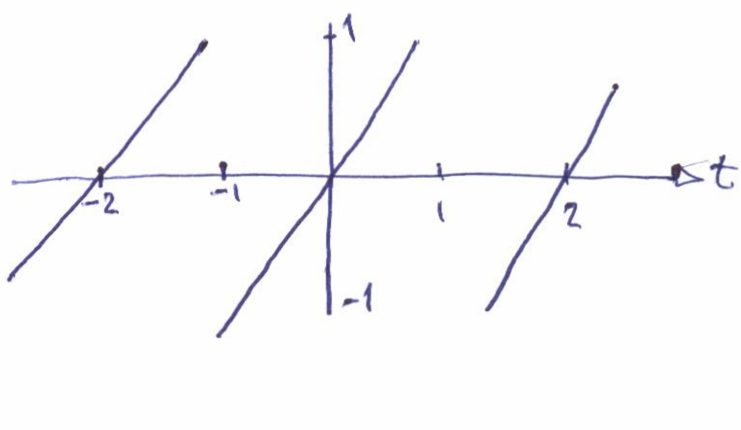
$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-1}^{0} 2^2 dt + \int_{0}^{\infty} 4e^{-t} dt$$

$$= 4[t]_{-1}^{0} + 4[-e^{-t}]_{0}^{\infty} = 4[0+1] + 4[0+1] = 8$$

EX.2 you have $g(t) = t \quad -\infty < t < \infty$ with period 2 seconds. Determine the content of this signal and sketch it.

solution since it has period $T = 2$ seconds, hence, it is a power signal.

$$P_g = \frac{1}{2}\int_{-1}^{1} g^2(t)\, dt = \frac{1}{2}\int_{-1}^{1} t^2\, dt$$



$$= \frac{1}{2}\,\frac{1}{3}\{1+1\} = \frac{1}{3}$$

Ex. 3 compute the rms values for each of the following signals,

(a) $g(t) = C\cos(\omega_0 t + \theta)$, (b) $k(t) = C_1\cos(\omega_1 t + \theta_1) + C_2\cos(\omega_2 t + \theta_2)$ where $\omega_1 \neq \omega_2$, and (c) $h(t) = D e^{j\omega_0 t}$.

Solution (a) $g(t) = C\cos(\omega_0 t + \theta)$ is periodic with $T_0 = \frac{2\pi}{\omega_0}$.

thus, it is a power signal

$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} C^2 \cos^2(\omega_0 t + \theta)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{C^2}{2}\left[1 + \cos(2\omega_0 t + 2\theta)\right] dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\frac{C^2}{2}\int_{-T/2}^{T/2} dt + \underbrace{\frac{C^2}{2T}\int_{-T/2}^{T/2} \cos(2\omega_0 t + 2\theta)\, dt}_{\text{zero}}$$

$$= \frac{C^2}{2T}\left[\frac{T}{2} + \frac{T}{2}\right] = \frac{C^2}{2T}T = \frac{C^2}{2}$$

$$\therefore \ rms_g = \frac{C}{\sqrt{2}}$$

solution (b) $x(t) = c_1 \cos(\omega_1 t + \theta_1) + c_2 \cos(\omega_2 t + \theta_2)$

where $\omega_1 \neq \omega_2$

this signal is a power signal by assuming $\frac{\omega_1}{\omega_2}$ is a rational number.

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} [c_1 \cos(\omega_1 t + \theta_1) + c_2 \cos(\omega_2 t + \theta_2)]^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} c_1^2 \cos^2(\omega_1 t + \theta_1) \, dt + \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} c_2^2 \cos(\omega_2 t + \theta_2) \, dt$$

$$+ \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \underbrace{2 c_1 c_2 \cos(\omega_1 t + \theta_1) \cos(\omega_2 t + \theta_2)}_{\to \text{zero}} \, dt$$

$$= \frac{c_1^2}{2} + \frac{c_2^2}{2}$$

$\therefore$ rms of $P_x \Rightarrow \text{rms} = \sqrt{P_x} = \sqrt{\frac{c_1^2}{2} + \frac{c_2^2}{2}}$

<u>solution</u> (c) $h(t) = D\,e^{j\omega_0 t}$

This signal is sinusoidal complex signal, then, it is a power signal.

$$P_h = \frac{1}{T_0}\int_0^{T_0} \left|D\,e^{j\omega_0 t}\right|^2 dt$$

$$= \frac{|D|^2}{T_0}\int_0^{T_0} \overset{1}{\cancel{\left|e^{j\omega_0 t}\right|^2}}\, dt$$

$$= \frac{|D|^2}{T_0}\int_0^{T_0} dt$$

$$= |D|^2 = D^2$$

$$\therefore\ rms_p = D$$

Signals and their Operations

□ Time Shifting

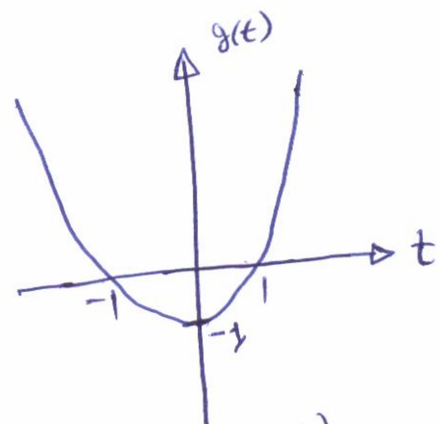
* The signal $f(t)$ can be advanced in time as

$$f(t) \xrightarrow{\text{advance by } t_0} \underbrace{f(t+t_0)}_{\text{went to Left}} \qquad (41)$$

* The signal can be delayed in time as

$$f(t) \xrightarrow{\text{delay by } t_0} \underbrace{f(t-t_0)}_{\text{went to right}} \qquad (42)$$

Ex.1 consider the signal $g(t)=t^2-1$, shift $g(t)$ to the right by one second, then shift it to the left by 2 seconds.

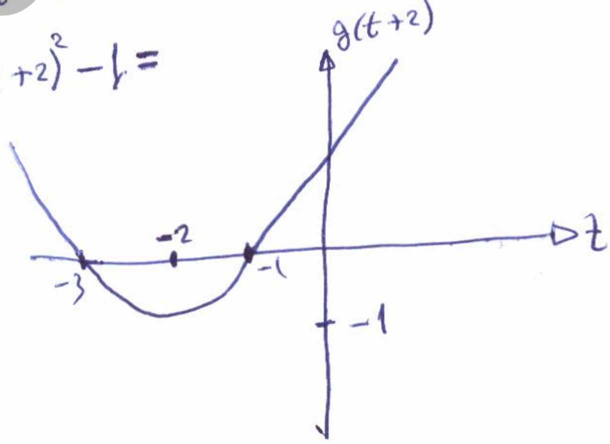


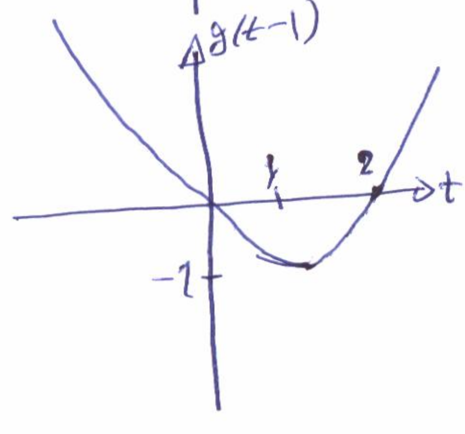
Solution (i) shift to the right by 1 s

$$g(t-1) = (t-1)^2 - 1 = t^2+1-2t-1 = t^2-2t$$

(ii) shift to the left by 2 s

$$g(t+2) = (t+2)^2 - 1 =$$





Amplitude scaling

* amplitude scaling stands for giving a gain to the amplitude of the signal

$$g(t) = A f(t) \quad \text{———} \quad (43)$$

* Amplitude scaling will not change the time.

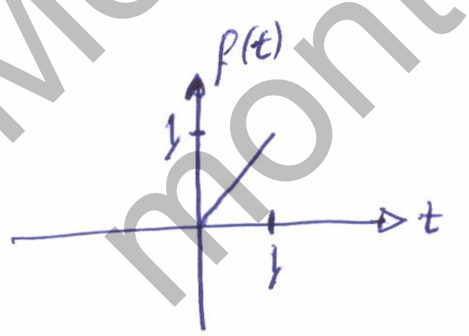
EX. 1 $f(t) = \sin(\omega_o t)$ can be amplitude scaled by $A = 5$ then

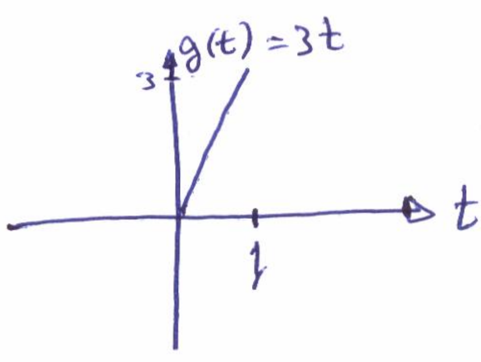
$g(t) = 5 f(t) = 5 \sin(\omega_o t)$.

EX. 2 $f(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$, sketch and ~~st~~ scale $f(t)$ by 3 units.

Solution let $g(t) = 3 f(t)$ amplitude scaling.

$= 3t \quad 0 \leq t \leq 1$ and zero otherwise





□ time scaling

* time scaling of a signal means moving the time points to the left or to the right by a certain seconds.

* In other words, Compression or expanssion.

* In the Compression: let $f(t)$ be a signal, then

$$g(t) = f(\alpha t) \qquad (44)$$

where $|\alpha| > 1$

* hence, Compression represents speeding-up the time by the factor α.

* In the expanssion:

$$g(t) = f(\alpha t) \qquad (44)$$

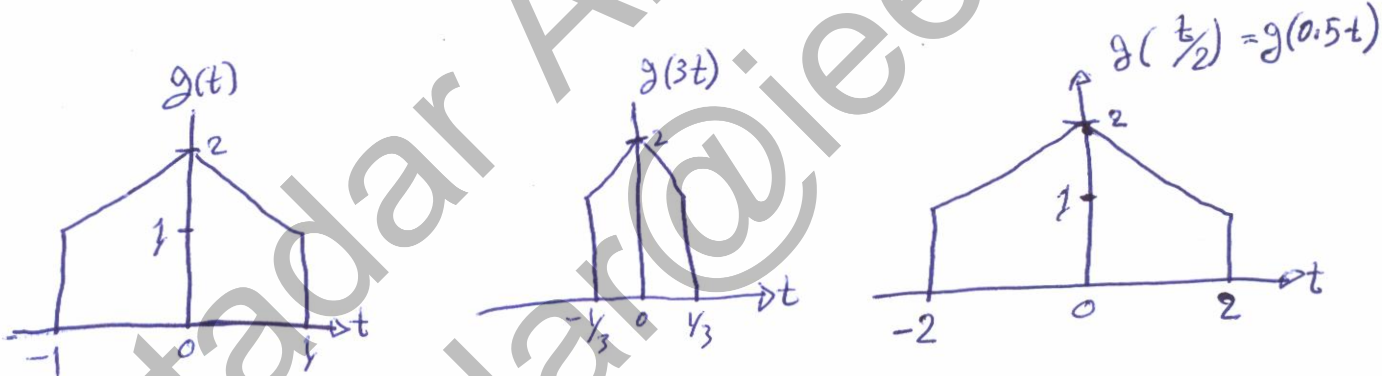
where $|\alpha| < 1$

* then, expanssion represents slowing-down the time by the factor α.

* In other words :

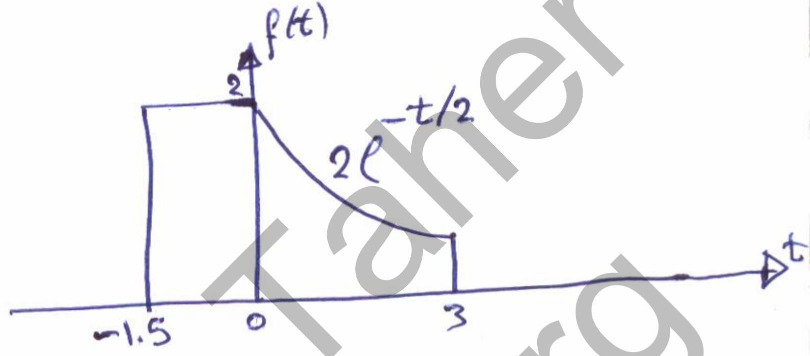
① if $|\alpha| > 1$ replace each t by $\frac{t}{\alpha}$ for compression (speeding-up) or shrinking.

② if $|\alpha| < 1$ replace each t by αt for expanssion (slowing-down).



$g(t)$ $\qquad$ $\alpha = 3$ $\qquad$ $\alpha = 0.5$

EX. For the signal shown below, describe it mathematically, and compress it by the factor 3 then expand it by factor 2 and sketch them all.

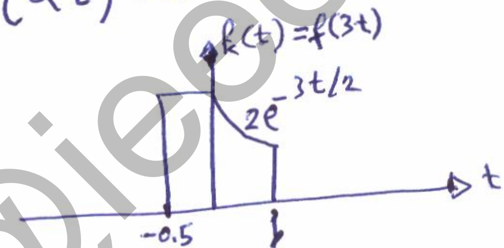


Solution

① mathematically

$$f(t) = \begin{cases} 2 & -1.5 \leq t < 0 \\ 2e^{-t/2} & 0 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}$$

② compress by 3 $\qquad f_c(t) = f(3t) = f(\alpha t) \Rightarrow \alpha = 3 > 1$



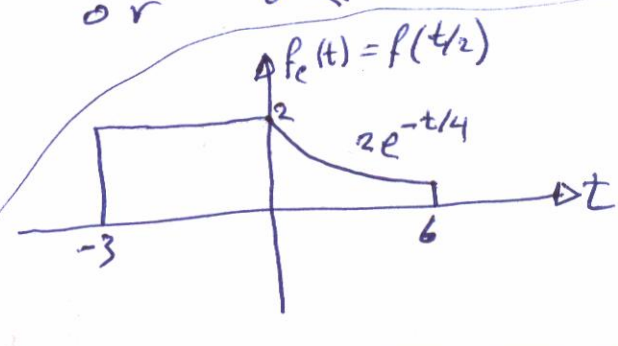
t	$t/3$
-1.5	-0.5
0	0
3	1

$$f_c(t) = \begin{cases} 2 & -1.5 \leq 3t < 0 \quad \text{or} \quad -0.5 \leq t < 0 \\ 2e^{-3t/2} & 0 \leq 3t < 3 \quad \text{or} \quad 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

③ Expansion by factor 2 $\qquad f_e(t) = f(\frac{t}{2})$

$$f_e(t) = f(\tfrac{t}{2})$$

$$f_e(t) = \begin{cases} 2 & -1.5 \leq \frac{t}{2} < 0 \quad \text{or} \quad -3 \leq t < 0 \\ 2e^{-t/4} & 0 \leq \frac{t}{2} < 3 \quad \text{or} \quad 0 \leq t < 6 \\ 0 & \text{otherwise} \end{cases}$$

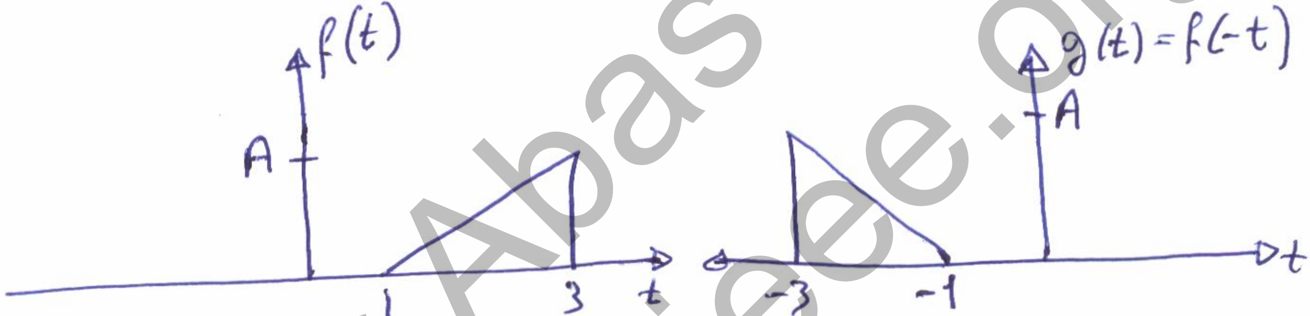


Time inversion (Reversal)

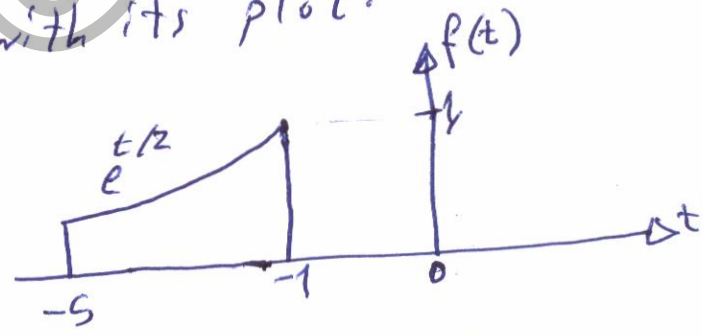
* Time inversion or time reversal or flipping, or reflection.

* if the signal is $f(t)$ then its reversal in time,

$$g(t) = f(-t) \qquad (45)$$



EX. Describe mathematically the signal shown below and find its time inversion with its plot.



Solution

$$f(t) = \begin{cases} e^{t/2} & -1 \geq t > -5 \\ 0 & \text{otherwise} \end{cases}$$

$$g(t) = f(-t) = \begin{cases} e^{-t/2} & -1 \geq t > -5 \text{ or } 1 \leq t < 5 \\ 0 & \text{otherwise} \end{cases}$$